Physikalisch-Technische Bundesanstalt
Braunschweig and Berlin
National Metrology Institute

RAD13-380

XIII RAD CONFERENCE

THIRTEENTH INTERNATIONAL CONFERENCE ON RADIATION, NATURAL SCIENCES, MEDICINE, ENGINEERING, TECHNOLOGY AND ECOLOGY
JUNE 16-20, 2025
HUNGUEST HOTEL SUN RESORT, HERCEG NOVI, MONTENEGRO

Defined solid-angle alpha spectrometry: Geometrical sensitivities in absolute Rn-222 activity measurements

Behnam Khanbabaee, Anja Honig, Stefan Röttger
PTB, Working Group 6.13 "Alpha and Gamma Spectrometry"

Contents

Introduction

Radon-222 (Rn-222): Main source of natural radiation and lung cancer risk.

Metrology challenge: Accurate measurement of the absolute activity of Rn-222 with low uncertainty

Method: Defined Solid Angle (DSA) alpha spectrometry

→Cryogenic radon deposition

→The activity A of the solidified radon source is calculated:

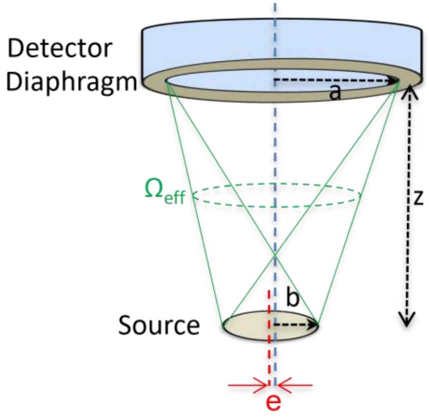
$$A = \frac{Counting\ rate}{G}$$

→The detection efficiency is determined by geometry factor G:

$$G = \frac{\Omega_{eff}}{4\pi} \quad (\Omega_{eff} = \text{effective solid angle})$$

→The accurate determination of the Ω_{eff} is crucial

→ Ω_{eff} depends on geometric parameters (see figure)



Common geometry for Defined Solid Angle (DSA) alpha spectrometry.
a: radius of the diaphragm,
b: radius of the source,
z: source-diaphragm distance,
e: eccentricity

Objectives

Goal

Optimize PTB's DSA-based primary standard for absolute radon activity.

Approach

Analyze how the accuracy of the geometry factor *G* depends on geometric parameters: *a*, *z*, *b*, and *e* (see figure).

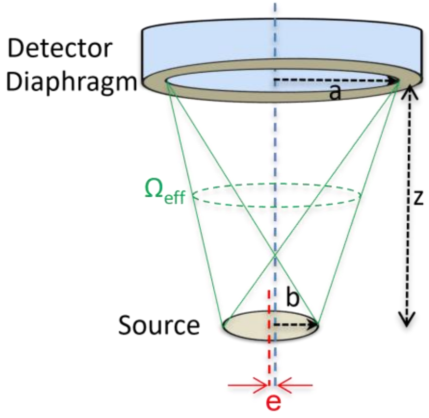
Steps

1. Calculate *G* using the Knoll and Curtis models.

2. Evaluate eccentricity effects using the Curtis model.

3. Identify critical geometric parameters.

4. Precisely measure these parameters to minimize the uncertainty of *G*.



Common geometry for Defined Solid Angle (DSA) alpha spectrometry: $G = \frac{\Omega_{eff}}{4\pi}$ (Ω_{eff} = effective solid angle)

Contents

Geometry factor determination using Knoll extension

The geometry factor (G) describes how many particles from a source can enter a detector, depending on the spatial arrangement:

$$G = \frac{\Omega_{eff}}{4\pi}$$

To determine the effective solid angle (Ω_{eff}) using Knoll extension:

$$\Omega_{eff} \cong 2\pi[1 - \frac{1}{(1+\beta)^{\frac{1}{2}}} - \frac{3}{8}\frac{\alpha\beta}{(1+\beta)^{\frac{5}{2}}} + \alpha^2[F1] - \alpha^3[F2]]$$

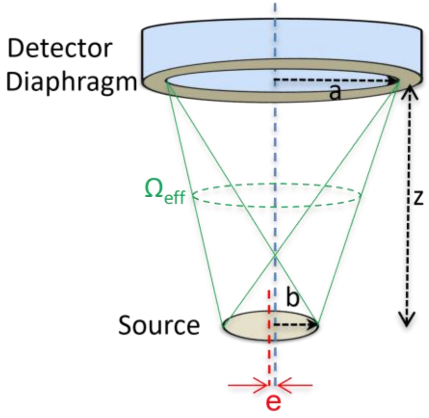
where:

$\alpha = (\frac{b}{z})^2$ and $\beta = (\frac{a}{z})^2$

with:

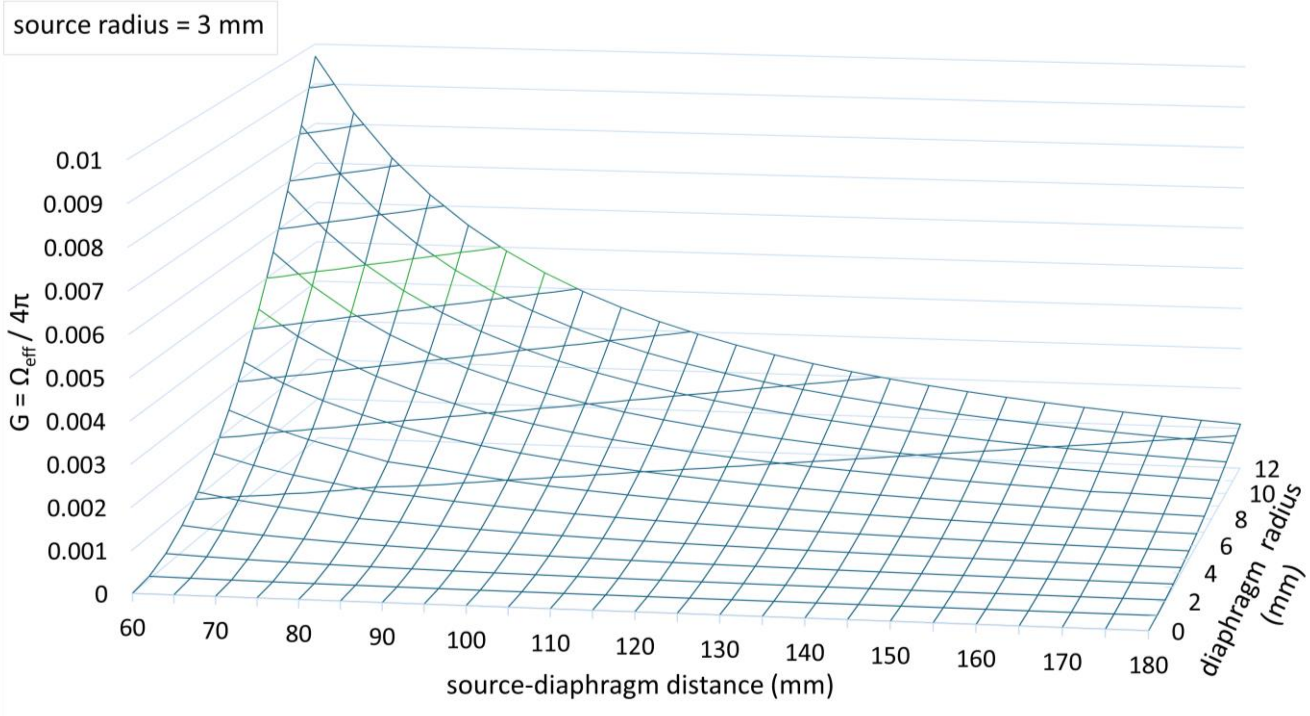
$F1 = \frac{5}{16}\frac{\beta}{(1+\beta)^{\frac{7}{2}}} - \frac{35}{64}\frac{\beta^2}{(1+\beta)^{\frac{9}{2}}}$ and

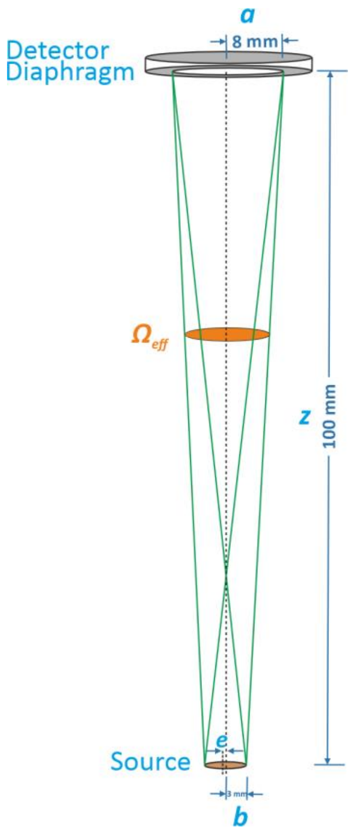
$$F2 = \frac{35}{128}\frac{\beta}{(1+\beta)^{\frac{9}{2}}} - \frac{315}{256}\frac{\beta^2}{(1+\beta)^{\frac{11}{2}}} + \frac{1155}{1024}\frac{\beta^3}{(1+\beta)^{\frac{13}{2}}}$$



Common geometry for Defined Solid Angle (DSA) alpha spectrometry.
a: radius of the diaphragm,
b: radius of the source,
z: source-diaphragm distance,
e: eccentricity

Geometry factor determination using Knoll extension

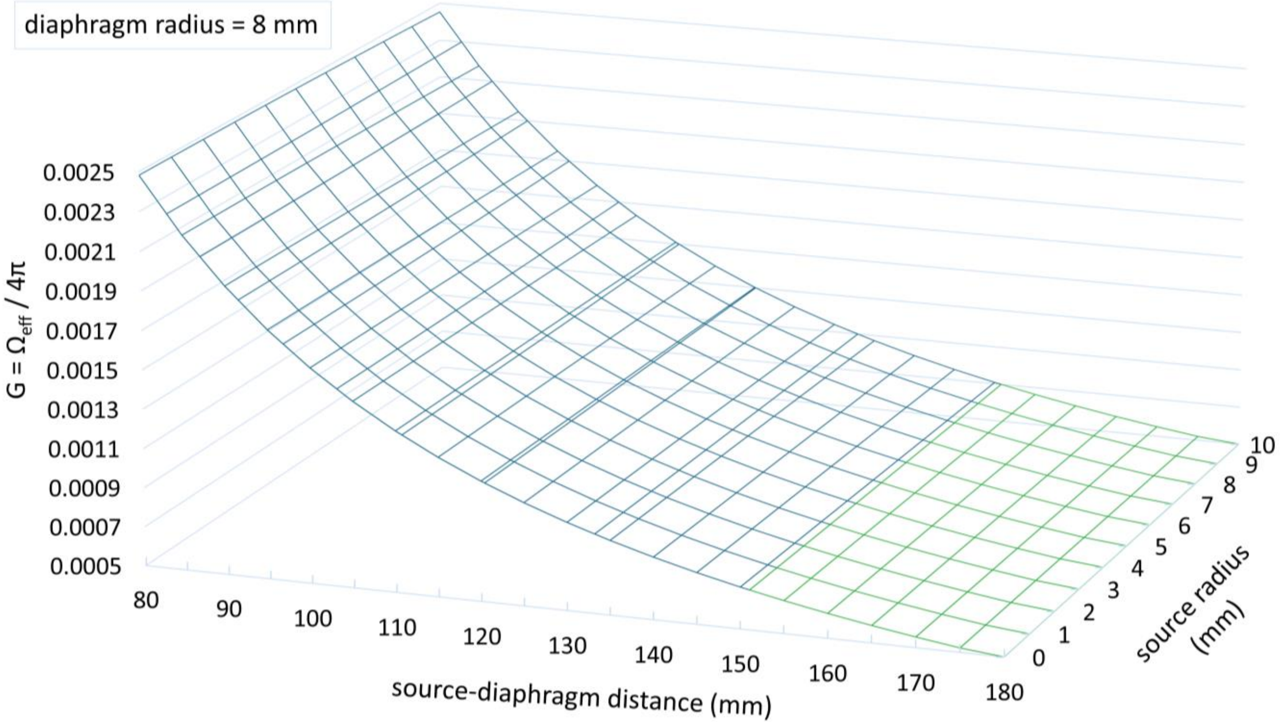


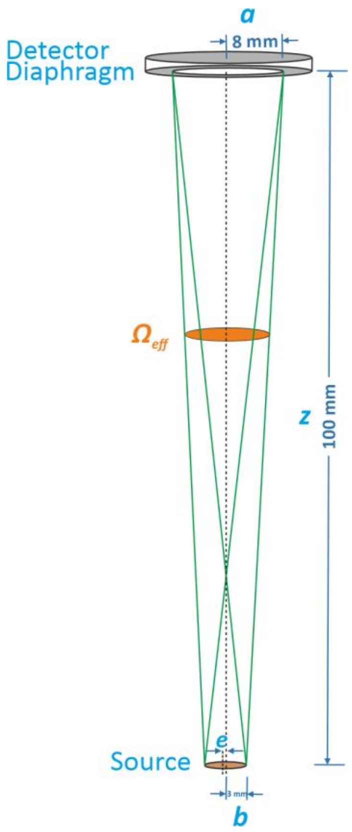


Geometry factor *G* vs. distance *z* and diaphragm radius *a*

With *b* = 3 mm, the Knoll extension shows that both *z* and *a* strongly influence *G*.

Geometry factor determination using Knoll extension

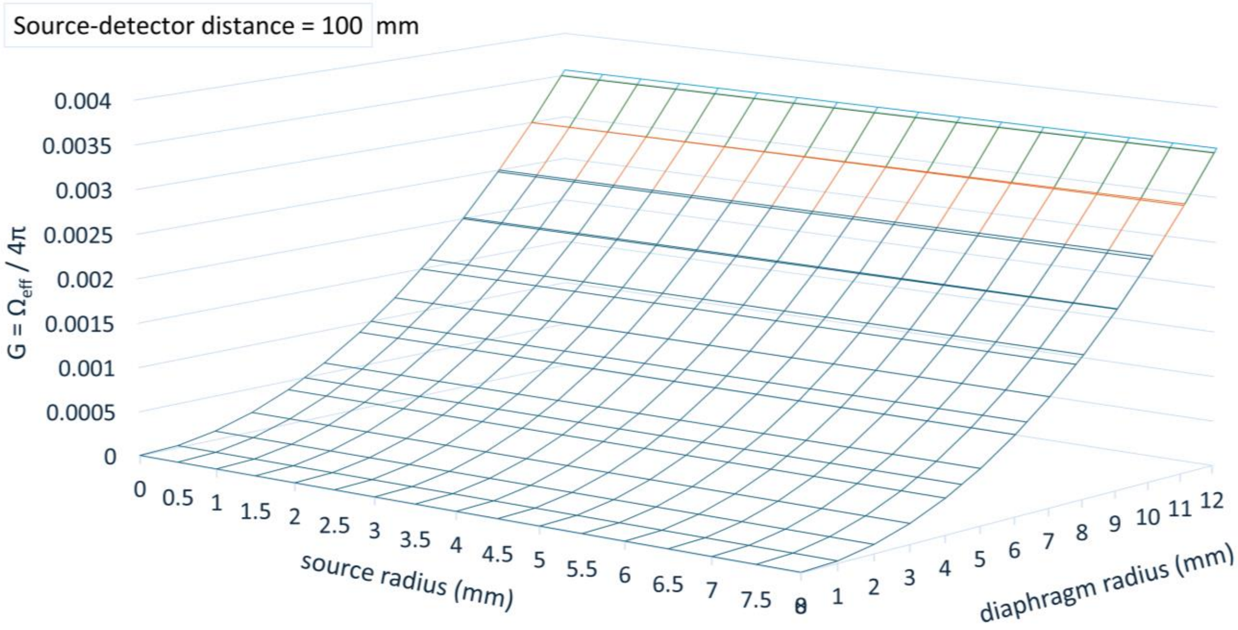


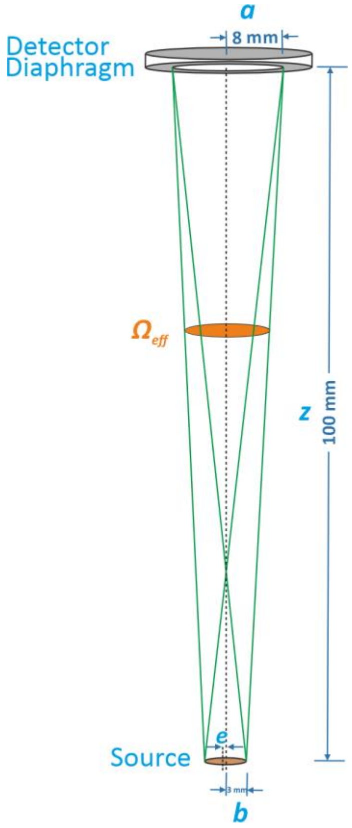


Geometry factor *G* vs. distance *z* and source radius *b*

The Knoll extension shows that *G* strongly depends on *z*, while variations in *b* have minimal impact.

Geometry factor determination using Knoll extension





Geometry factor *G* vs. source radius *b* and diaphragm radius *a*

At z = 100 mm, the Knoll extension shows that G depends strongly on diaphragm radius a, rising sharply up to ~0.004 at a = 12 mm due to increased solid angle. In contrast, varying b from 0 mm to 8 mm has negligible impact, confirming its minor influence in this setup.

Contents

Geometry factor determination using Curtis extension

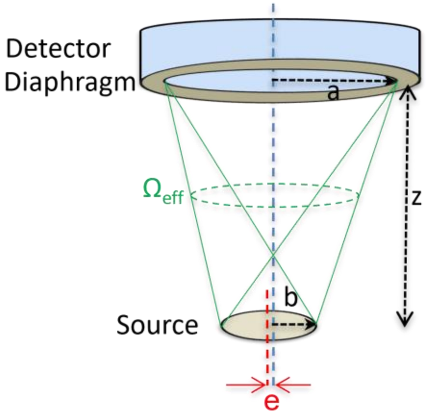
Considering eccentricity in solid angle calculation

When the source and diaphragm axes are misaligned by an eccentricity e, the geometry becomes more complex.

In such cases, the effective solid angle Ω_{eff} can be calculated using the Curtis equation.

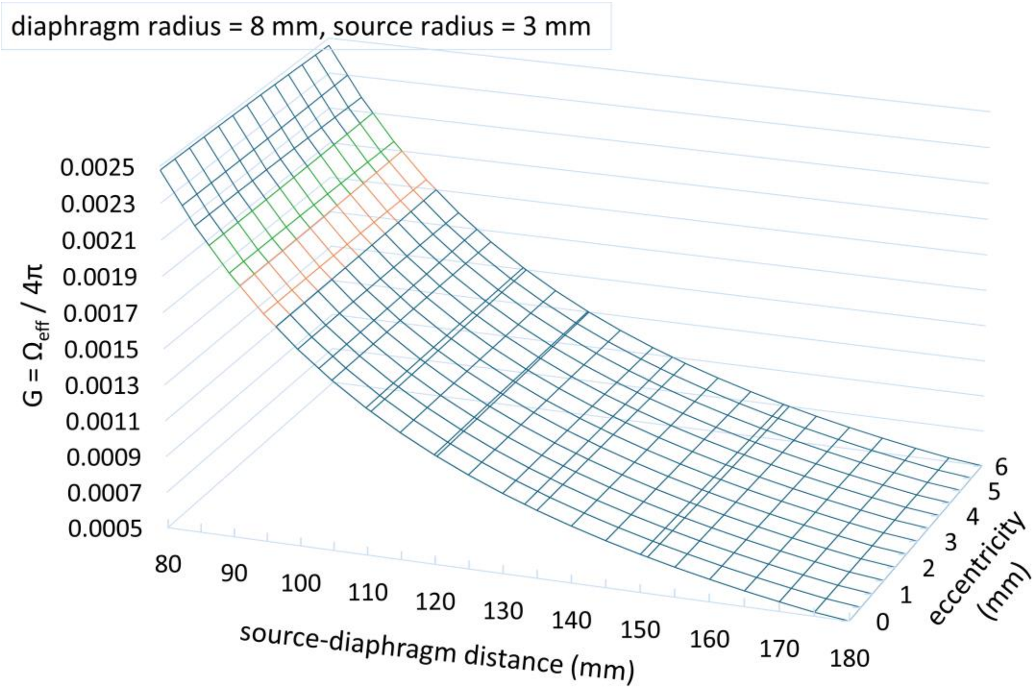
$$G = \frac{\Omega_{eff}}{4\pi}$$

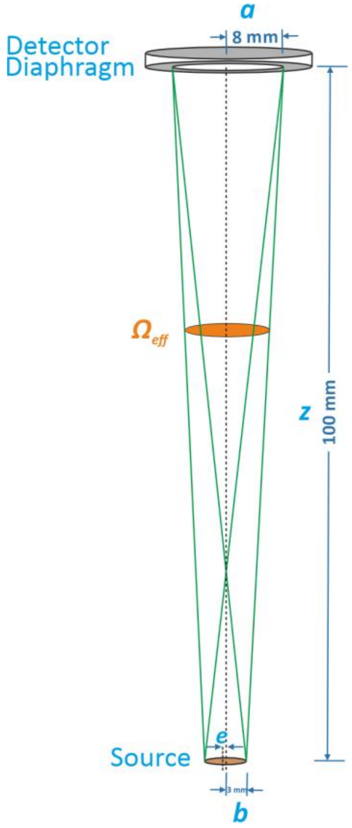
$$\Omega_{eff} = \pi \frac{a^2}{z^2}\left[1 - \frac{3}{4}\frac{(a^2+b^2+2e^2)}{z^2} + \frac{5}{8}\frac{(a^4+b^4+3e^4+6a^2e^2+6b^2e^2+3a^2b^2)}{z^4} - \frac{35}{64}\frac{(a^6+b^6+4e^6+6a^2b^4+6a^4b^2+12a^4e^2+12b^4e^2+18a^2e^4+18b^2e^4+36a^2b^2e^2)}{z^6}\right.$$



Common geometry for Defined Solid Angle (DSA) alpha spectrometry.
a: radius of the diaphragm,
b: radius of the source,
z: source-diaphragm distance,
e: eccentricity

Geometry factor determination using Curtis extension

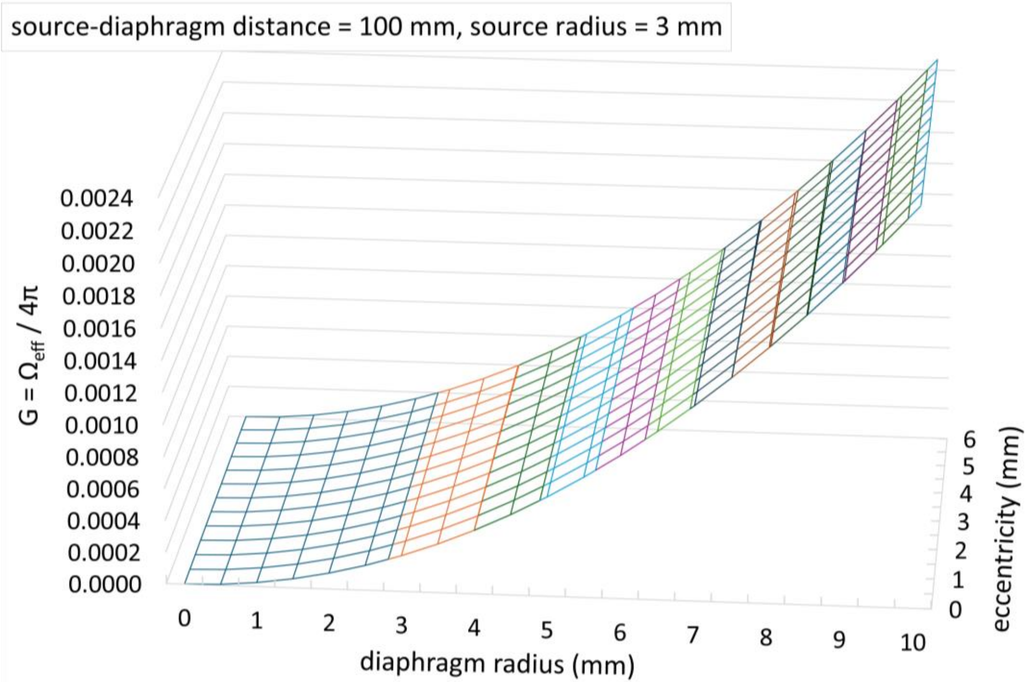


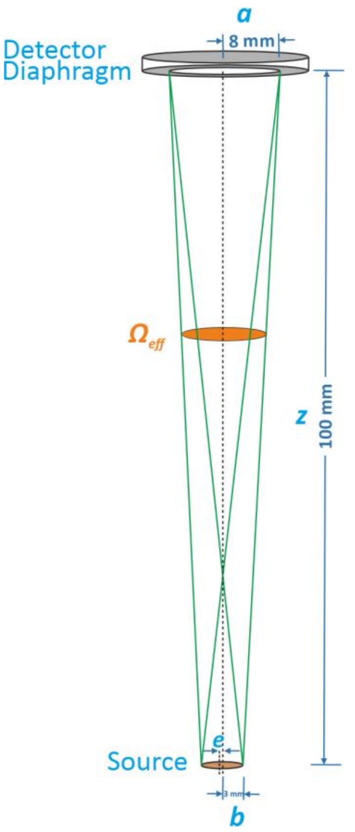


Geometry factor *G* vs. distance *z* and eccentricity *e*

The Curtis formulation shows that *G* depends strongly on distance *z*, while its dependence on eccentricity *e* from 1 mm to 6 mm is minimal when other parameters are held constant.

Geometry factor determination using Curtis extension



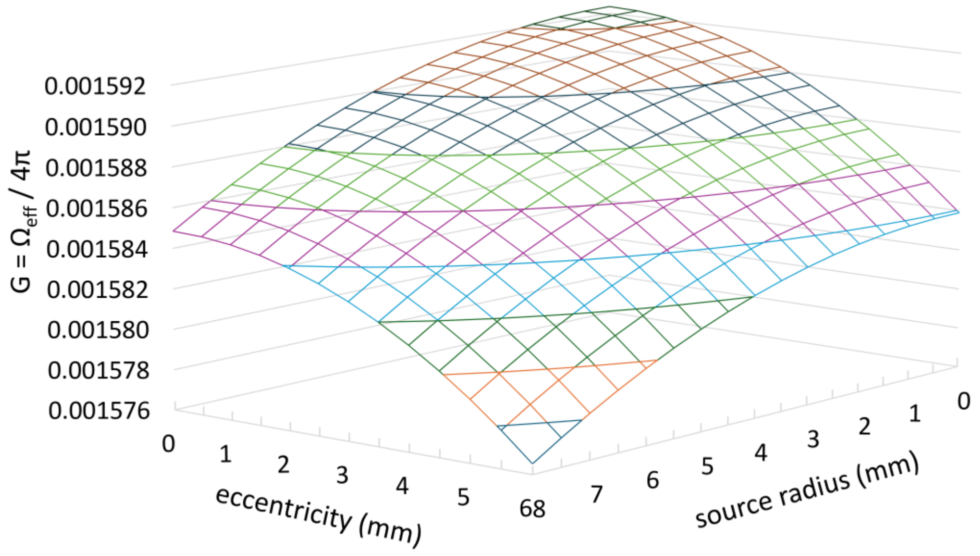


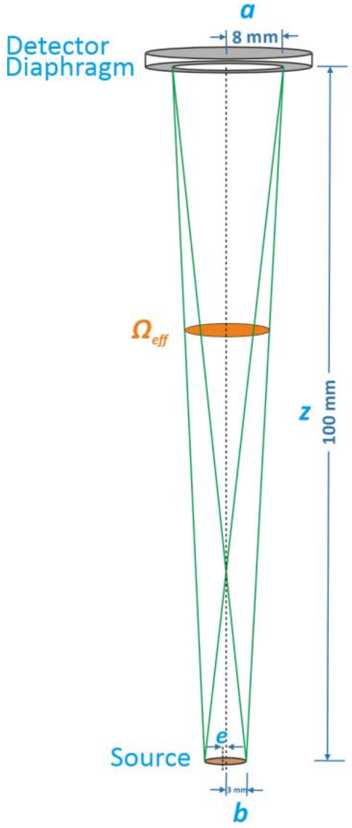
Geometry factor *G* vs. diaphragm radius *a* and eccentricity *e*

The Curtis formulation shows that *G* increases from nearly zero to 0.0024 with increasing diaphragm radius *a* , keeping *z* = 100 mm and *b* = 3 mm constant. Thus, *a* is a crucial factor for *G*.

Geometry factor determination using Curtis extension

source-diaphragm distance = 100 mm, diaphragm radius = 8 mm





Geometry factor *G* vs. source radius *b* and eccentricity *e*

For constant values of z = 100 mm and a = 8 mm, G decreases slightly from 0.001592 to 0.001576 as both b and e increase. This confirms their small influence in this setup.

Contents

Comparison with the reported geometry factors

Comparison of geometry factors reported by National Metrology Institutes (NMIs) with calculated values

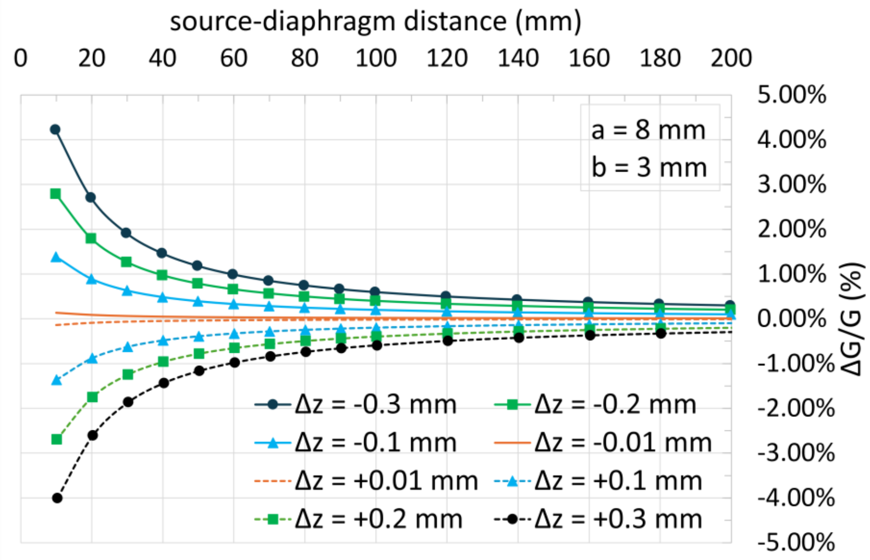
The values calculated using the Knoll and Curtis extensions largely agree with the geometry factors reported by NMIs and show only minimal relative deviation (see table). Both methods produce consistent results, confirming their reliability.

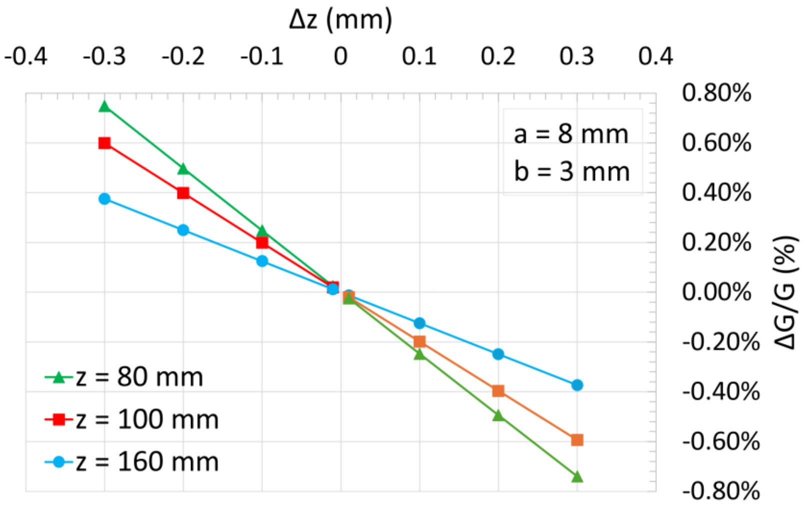
NMI	reported G	calculated G using Knoll extension	$\Delta G/G$ [%]	calculated G using Curtis extension (e = 0)	$\Delta G/G$ [%]	calculated G using Curtis extension ($e \neq 0$)[1]	$\Delta G/G$ [%]
NLE-LNHB	2.019×10^{-3}	2.019×10^{-3}	-0.013	2.019×10^{-3}	-0.013	2.018×10^{-3} (e = 1 mm)	-0.028
PTB	$(5.625 \pm 0.010) \times 10^{-4}$	5.626×10^{-4}	0.016	5.626×10^{-4}	0.016		
METAS	$8.927(18) \times 10^{-4}$	8.928×10^{-4}	0.009	8.928×10^{-4}	0.011	8.928×10^{-4} (e = 0.3 mm)	0.009
KRISS	6.019×10^{-4}	6.019×10^{-4}	0.000	6.019×10^{-4}	0.000	6.019×10^{-4} (e = 0.5 mm)	-0.005
KRISS	9.381×10^{-4}	9.382×10^{-4}	0.010	9.382×10^{-4}	0.010	9.381×10^{-4} (e = 0.5 mm)	0.004
KRISS	1.348×10^{-3}	1.348×10^{-3}	0.007	1.348×10^{-3}	0.007	1.348×10^{-3} (e = 0.5 mm)	0.002
NMI-China	1.2594×10^{-3}	1.2594×10^{-3}	0.001	1.2594×10^{-3}	0.000		

[1]The chosen eccentricity value was based on reported data from NMIs.

Contents

Sensitivity of the geometry factor to distance





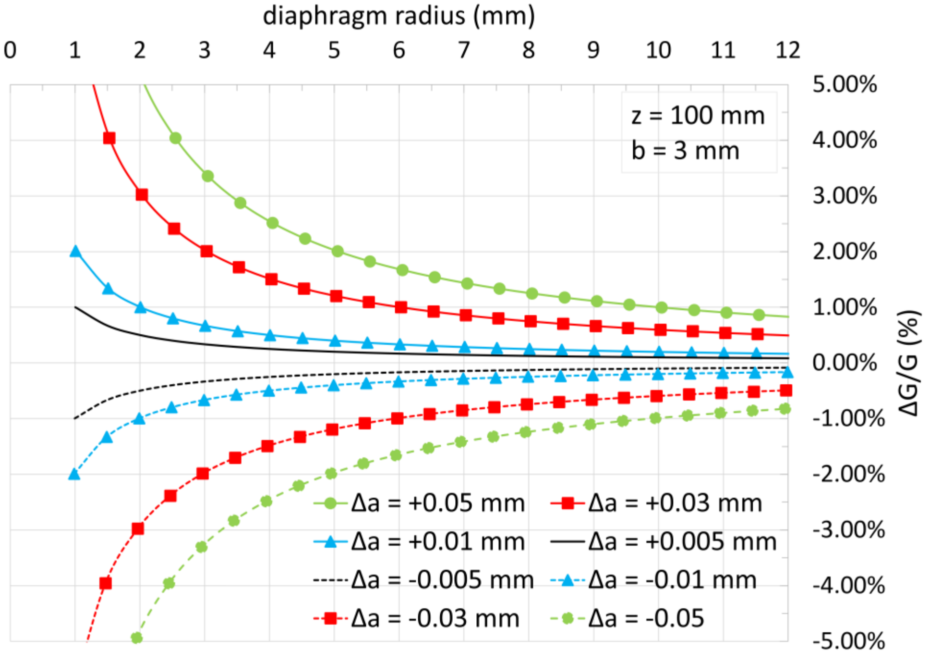
Relative variation of the Δ*G*/*G* vs. distance *z*

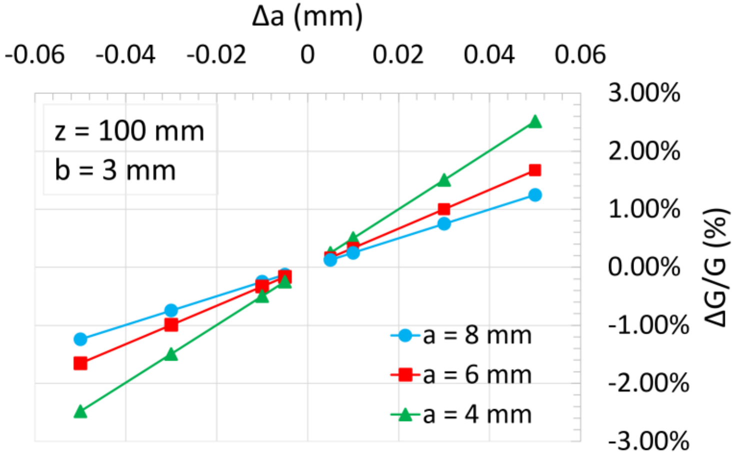
Using the Knoll extension, Δ*G*/*G* increases sharply at short distances and exhibits nonlinear behavior. Larger Δ*z* values lead to greater changes in *G*.

Relative variation of Δ*G*/*G* vs. Δ*z* at different *z* values

At *z* = 80 mm, 100 mm and 160 mm, the same modulation Δ*z* has a smaller effect on Δ*G*/*G* as *z* increases, indicating lower sensitivity at larger distances.

Sensitivity of the geometry factor to diaphragm radius





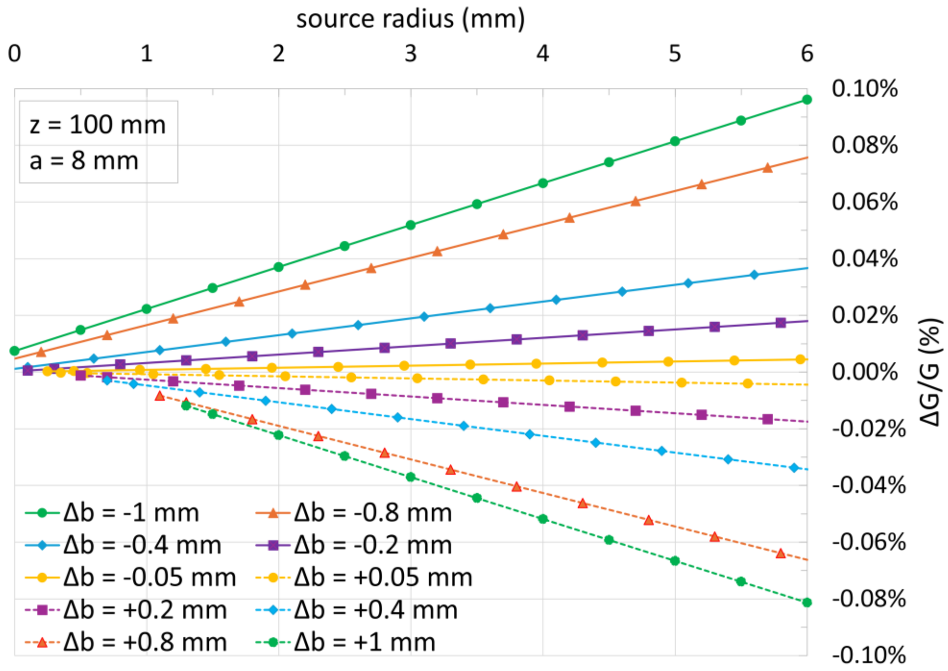
Relative variation of the Δ*G*/*G* vs. diaphragm radius *a*

Using the Knoll extension, Δ*G*/*G* increases rapidly at smaller values of *a*. Larger Δ*a* leads to greater changes in *G*.

Relative variation of Δ*G*/*G* vs. Δ*a* at different *a* values

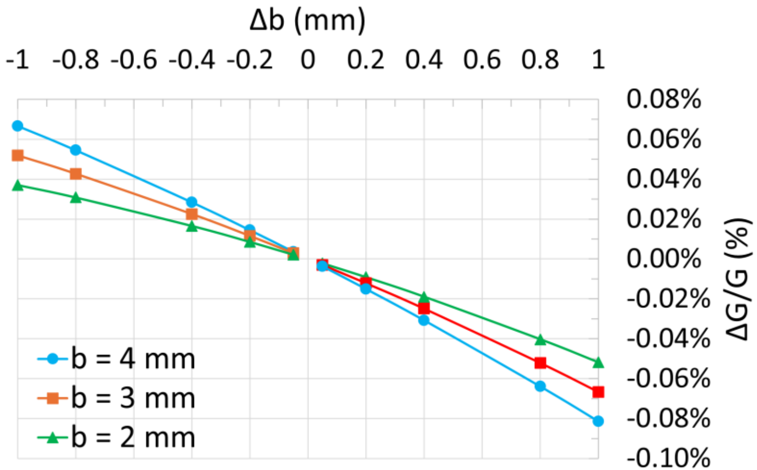
At *a* = 4 mm, 6 mm and 8 mm, the same modulation Δ*a* results in a smaller Δ*G*/*G* as *a* increases, indicating lower sensitivity at larger diaphragm radii.

Sensitivity of the geometry factor to source radius



Relative variation of the Δ*G*/*G* vs. source radius *b*

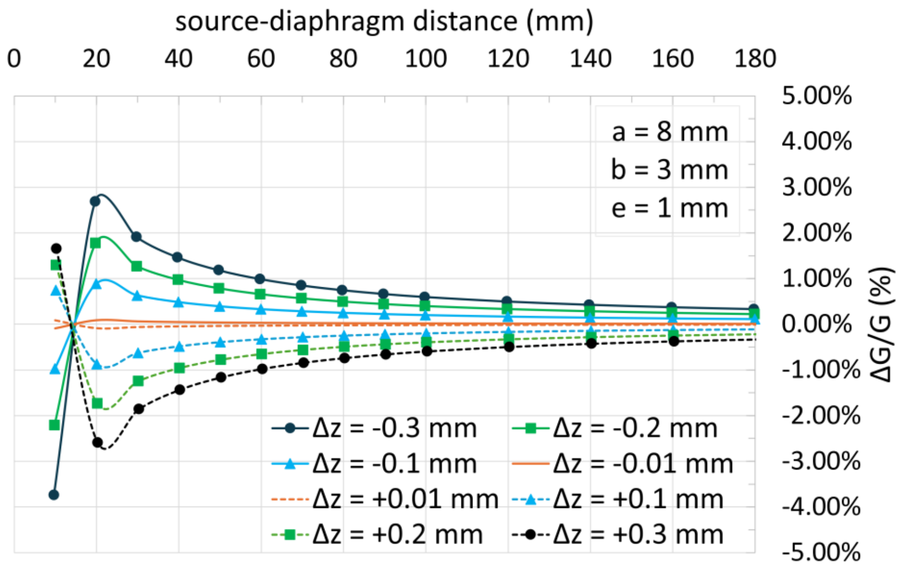
Using the Knoll extension, Δ*G*/*G* increases linearly with *b*, and larger Δ*b* values yield greater changes in the geometry factor.

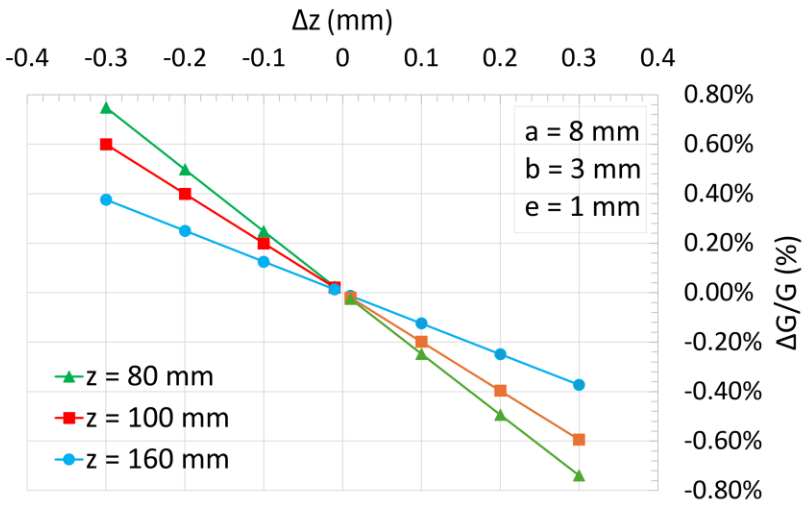


Relative variation of ΔG/G vs. Δ*b* at different *b* values

At *b* = 2 mm, 3 mm and 4 mm, the same modulation Δ*b* results in a smaller Δ*G*/*G* as *b* decrease, indicating lower sensitivity at smaller source radii.

Sensitivity of the geometry factor to distance





Relative variation of the Δ*G*/*G* vs. distance *z*

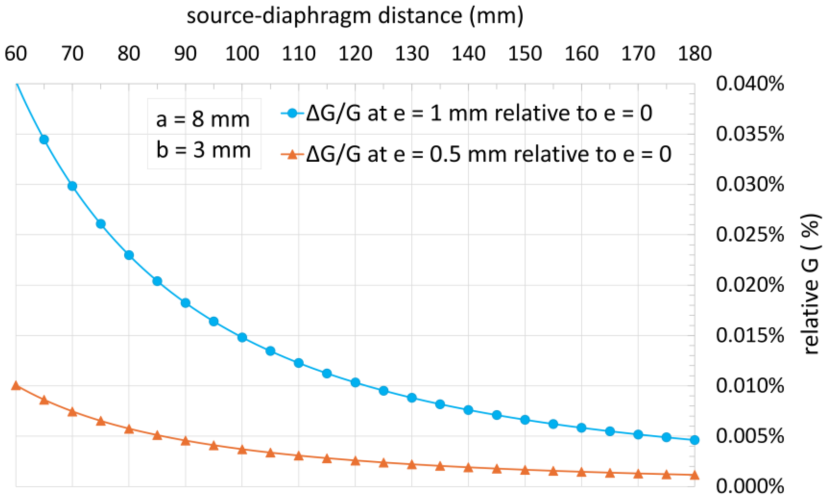
Using the Curtis extension (*e* = 0), Δ*G*/*G* increases sharply at short distances and shows nonvalid behavior.

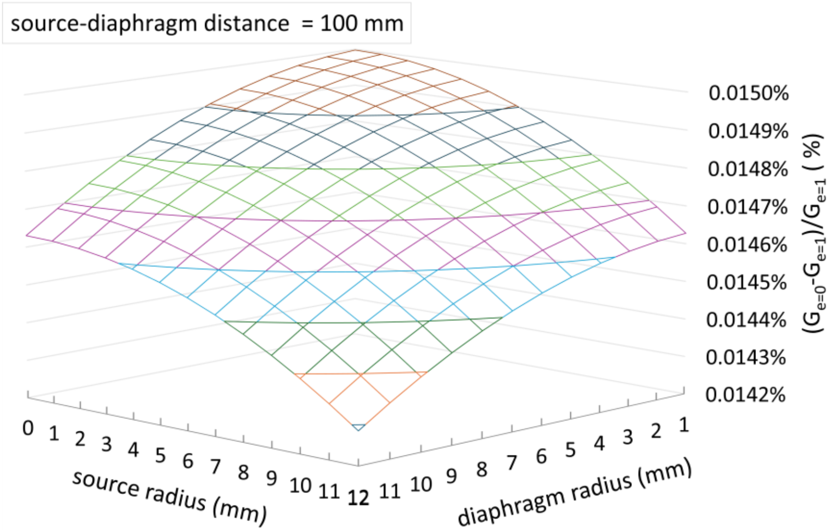
Relative variation of ΔG/G vs. Δ*z* at different *z* values

The results demonstrate a high level of agreement between the Curtis and Knoll models under typical operating conditions (slide 18), thereby confirming the equivalence of the two approaches in applicable ranges.

Contents

Eccentricity effect on accuracy of the *G*





$\frac{G_{e=0}-G_{e=1}}{G_{e=1}}$ vs. distance *z* (blue dotted line) and $\frac{G_{e=0}-G_{e=0.5}}{G_{e=0.5}}$ vs. distance *z* (red triangle line) using the Curtis extension.

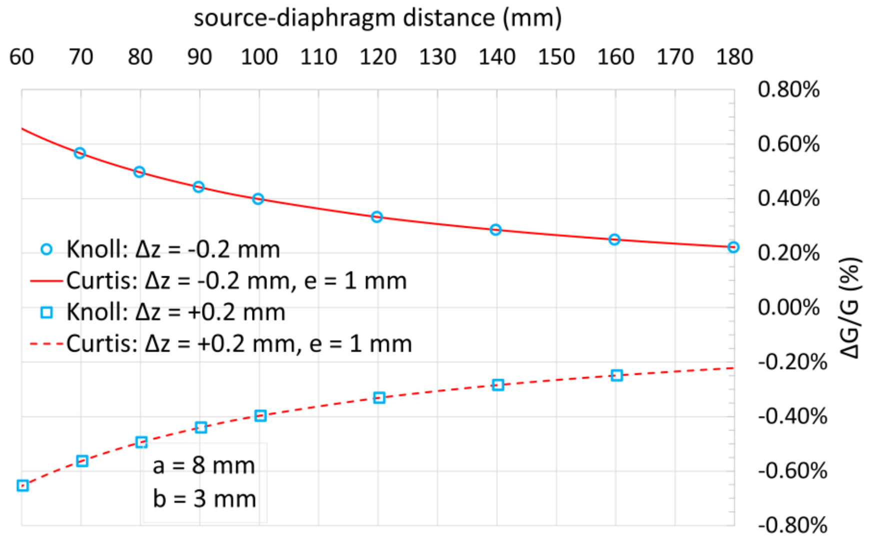
Within the practical range 0 mm ≤ *e* ≤ 1 mm, eccentricity has a negligible effect on the *G*.

$\frac{G_{e=0}-G_{e=1}}{G_{e=1}}$ vs. source radius *b* and diaphragm radius *a* using the Curtis extension.

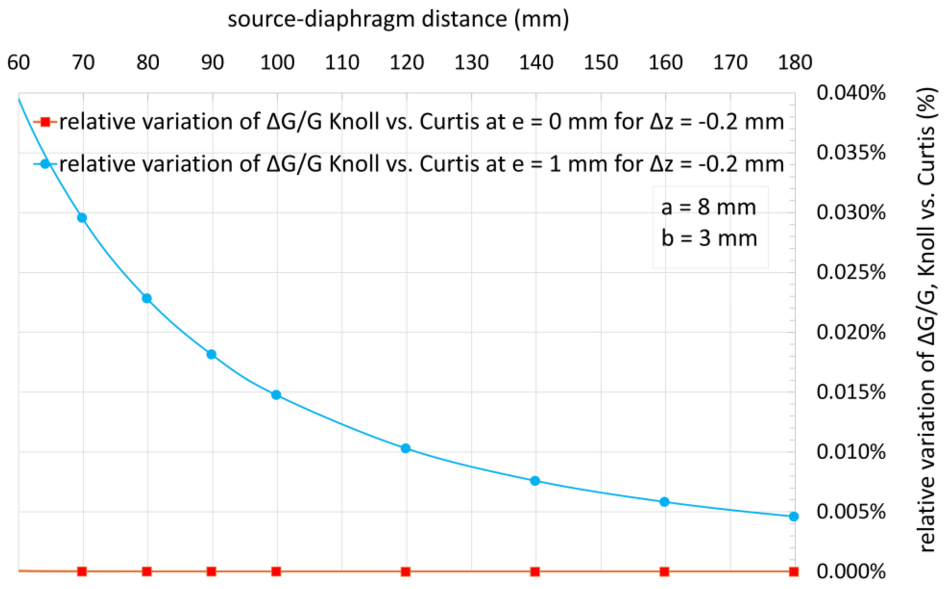
The variation decreases monotonically from ~0.0150 % to 0.0142 % as either radius increases from 0 mm to 12 mm, indicating that *G* becomes less sensitive to *e* for larger radii.

Contents

Comparison of Knoll and Curtis extensions



$\Delta G/G$ vs. distance z, calculated using the Knoll extension (open blue circles for nominal distances modified by Δz = −0.2 mm and open blue squares for Δz = +0.2 mm) with the Curtis extension (solid red line for Δz = −0.2 mm and dashed red line for Δz = +0.2 mm) at e = 1 mm .



Relative difference between calculated $\Delta G/G$ using the Knoll and Curtis extensions at e = 1 mm (blue dotted line), highlighting the impact of eccentricity.

Contents

Summary

This study numerically evaluated how the geometry factor *G* in Defined Solid Angle (DSA) alpha spectrometry depends on four key parameters (diaphragm radius *a*, source-diaphragm distance *z*, source radius *b*, and eccentricity *e*). Both the Knoll and Curtis formalisms were used. The analyses showed the following:

- **Diaphragm radius *a*** and **distance *z*** dominate *G*. Smaller *a* or larger *z* values drive *G* toward zero, while larger *a* and smaller *z* values maximize *G*.
- **The source radius *b*** has a negligible influence on *G* in the typical range of 1 mm to 5 mm.
- **The eccentricity *e*** (up to 1 mm) contributes only slightly (< 0.015 %) to *G* compared to *a* and *z*.
- **The Knoll and Curtis models** yield essentially identical *G* values for all practical geometries and differ only at unrealistically small distances.

A sensitivity analysis using small perturbations Δa, Δz and Δb quantified their relative effects: a change in *a* of ± 0.01 mm leads to a change in *G* of ~0.2 %, while achieving the same effect over *z* would requires a value of $\Delta z \approx \pm 0.1$ mm and over *b* a ten times larger Δb.

Conclusion

- To minimize the measurement uncertainty of absolute Rn-222 activity using Defined Solid Aangle (DSA) alpha spectrometry, National Metrology Institutes (NMIs) should prioritize the precise determination of the diaphragm radius a and source-diaphragm distance z.
- The source radius b and small misalignments (eccentricity e) have a significantly smaller influence on the geometry factor G.
- The Curtis and Knoll formalisms show high agreement under typical measurement conditions and are therefore interchangeable for practical purposes.
- The diaphragm radius a is critical for the accuracy of G, while eccentricity e is negligible in the range from 0 mm to 1 mm.

Physikalisch-Technische Bundesanstalt
Braunschweig and Berlin
Bundesallee 100
38116 Braunschweig
Germany

Dr. Behnam Khanbabaee
Telefon: 0531 5926160
E-Mail: Behnam.khanbabaee@ptb.de
www.ptb.de

Stand: 2025-06